## Process and Practice Standards for all Courses 9 - 12

| Content Area | Number | Content Area Topic |
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| Mathematics | 1 | Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole. |
| Mathematics | 2 | Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. |
| Mathematics | 3 | Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies and use various methods of proof. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |

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| Mathematics | 4 | Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |
| Mathematics | 5 | Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. Regarding technology, students use it strategically as a tool to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication, and problem solving. Note: Elementary students must learn how to fluently perform the basic arithmetic operations independent of the use of a calculator. |
| Mathematics | 6 | Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions including correct mathematical language in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. |
| Mathematics | 7 | Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. |

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| Mathematics | 8 | Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |

**MATHEMATICS: EIGHTH GRADE**

| Content Area | Grade Level/Span | Strand | Number | Content Area Topic |
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| Mathematics | 8 | Number Sense | 1 | Know that there are numbers that are rational and irrational and explain the difference between them. Give examples of rational and irrational numbers. Understand that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats into a rational number. |
| Mathematics | 8 | Number Sense | 2 | Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions involving irrational numbers. |
| Mathematics | 8 | Number Sense | 3 | Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 × 3^–5 = 3^–3 = 1/3^3 = 1/27. |
| Mathematics | 8 | Number Sense | 4 | Use square root and cube root symbols to represent solutions to equations of the form x^2 = p and x^3 = p, where p is a positive rational number. Evaluate square roots of perfect squares and cube roots of perfect cubes. |
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| Mathematics | 8 | Computation | 1 | Solve multi-step real-world problems involving addition, subtraction, multiplication, and division with rational numbers. |
| Mathematics | 8 | Computation | 2 | Solve real-world and mathematical problems involving numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Interpret scientific notation that has been generated by technology, such as, a scientific calculator, graphing calculator, and excel spreadsheet. |
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| Mathematics | 8 | Algebra and Functions | 1 | Fluently solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Represent real-world problems using linear equations and solve such problems. |
| Mathematics | 8 | Algebra and Functions | 2 | Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by transforming a given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). |
| Mathematics | 8 | Algebra and Functions | 3 | Construct a function to model a linear relationship between two quantities given a verbal description, table of values, or graph. Recognize in y = mx + b that m is the slope (rate of change) and b is the y-intercept of the graph and describe the meaning of each in the context of a problem. |
| Mathematics | 8 | Algebra and Functions | 4 | Compare two different linear relationships given in different forms (table of values, equation, verbal description, and graph). For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |
| Mathematics | 8 | Algebra and Functions | 5 | Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. |

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| Mathematics | 8 | Algebra and Functions | 6 | Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. |
| Mathematics | 8 | Algebra and Functions | 7 | Write a system of two linear equations that represents a real-world problem and solve the problem. |
| Mathematics | 8 | Algebra and Functions | 8 | Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. |
| Mathematics | 8 | Algebra and Functions | 9 | Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, has a maximum or minimum value). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. |
| Mathematics | 8 | Algebra and Functions | 10 | Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. Describe similarities and differences between linear and nonlinear functions from tables, graphs, verbal descriptions, and equations. |
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| Mathematics | 8 | Geometry and Measurement | 1 | Perform constructions with or without technology: angle and segment bisectors, copies of segments and angles, and perpendicular segments. Describe and justify the constructions. |
| Mathematics | 8 | Geometry and Measurement | 2 | Identify, define and describe attributes of three-dimensional geometric objects (right rectangular prisms, cylinders, cones, spheres, and pyramids) and describe the two-dimensional figure that results from slicing these objects. |
| Mathematics | 8 | Geometry and Measurement | 3 | Verify experimentally the properties of rotations, reflections, and translations; lines are mapped to lines, and line segments to line segments of the same length; angles are mapped to angles of the same measure; and parallel lines are mapped to parallel lines. |
| Mathematics | 8 | Geometry and Measurement | 4 | Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. |
| Mathematics | 8 | Geometry and Measurement | 5 | Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar figures, describe a sequence that exhibits the similarity between them. |
| Mathematics | 8 | Geometry and Measurement | 6 | Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. |
| Mathematics | 8 | Geometry and Measurement | 7 | Know facts about the angle sum and exterior angles of triangles, angles created when parallel lines are cut by a transversal (corresponding, alternate interior, alternate exterior, consecutive interior, consecutive exterior, vertical), and angle-angle criterion for similarity of triangles. Use this information to solve real-world and mathematical problems. |
| Mathematics | 8 | Geometry and Measurement | 8 | Explain the reasoning of a given proof of the Pythagorean Theorem and its converse. |
| Mathematics | 8 | Geometry and Measurement | 9 | Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two dimensions. |

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| Mathematics | 8 | Geometry and Measurement | 10 | Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. |
| Mathematics | 8 | Geometry and Measurement | 11 | Solve real-world and mathematical problems involving volume and surface area of cones, spheres, and pyramids. |
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| Mathematics | 8 | Data Analysis, Statistics, and Probability | 1 | Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 2 | Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and describe the model fit by judging the closeness of the data points to the line. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 3 | Write and use an equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and y-intercept. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 4 | Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 5 | Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Understand and use appropriate terminology to describe independent, dependent, complementary, and mutually exclusive events. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 6 | Represent sample spaces and find probabilities of compound events (independent and dependent) using methods such as organized lists, tables, and tree diagrams. |
| Mathematics | 8 | Data Analysis, Statistics, and Probability | 7 | For events with a large number of outcomes, understand the use of the Multiplication Counting Principle. Develop the Multiplication Counting Principle and apply it to situations with a large number of outcomes. |

# Algebra 1

| Content Area | Course | Strand | Number | Content Area Topic |
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| Mathematics | Algebra 1 | Number Sense | 1 | Understand the heirarchy and relationships of numbers and sets of numbers within the Real Number System. |
| Mathematics | Algebra 1 | Number Sense | 2 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. * |
| Mathematics | Algebra 1 | Number Sense | 3 | Rewrite and evaluate numeric expressions with positive rational exponents using the properties of exponents. |
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| Mathematics | Algebra 1 | Linear equations and inequalities | 1 | Fluently solve linear equations and inequalities in one variable. Explain and justify each step in solving an equation starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| Mathematics | Algebra 1 | Linear equations and inequalities | 2 | Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution as the original. Understand that similar logic applies to solving systems of equations simultaneously. |
| Mathematics | Algebra 1 | Linear equations and inequalities | 3 | Represent real-world problems using linear equations and inequalities and solve such problems. Interpret the solution(s) and determine if the solution(s) is reasonable. |
| Mathematics | Algebra 1 | Linear equations and inequalities | 4 | Solve equations and formulas for a specified variable including equations with coefficients represented by letters |
| Mathematics | Algebra 1 | Linear equations and inequalities | 5 | Solve compound linear inequalities using properties of order. |
| Mathematics | Algebra 1 | Functions | 6 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| Mathematics | Algebra 1 | Functions | 7 | Identify the domain and range of relations represented in tables, graphs, verbal descriptions, and equations. |
| Mathematics | Algebra 1 | Functions | 8 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Represent linear functions as graphs from equations. |
| Mathematics | Algebra 1 | Functions | 9 | Represent linear functions in real-world problems using tables, graphs, verbal descriptions, and equations. Translate fluently among tables, graphs, verbal descriptions, and equations. Determine and interpret the slope and intercepts of linear functions. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 1 | Functions | 10 | Translate among equivalent forms of equations for linear functions (i.e., slope-intercept, point-slope and standard). Recognize that different forms reveal more or less information about a given situation |

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| Mathematics | Algebra 1 | Functions | 11 | Graph a linear inequality in two variables to determine the solution set of the inequality. |
| Mathematics | Algebra 1 | Systems | 12 | Graph a pair of linear inequalities in two variables with and without technology to determine the solution set of the inequality. |
| Mathematics | Algebra 1 | Systems | 13 | Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Compare linear functions and exponential functions using tables, graphs and equations. |
| Mathematics | Algebra 1 | Systems | 14 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. |
| Mathematics | Algebra 1 | Systems | 15 | Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing (exact or approximate) , substitution or elimination. |
| Mathematics | Algebra 1 | Systems | 16 | Write a system of two linear equations that represents a real-world problem and solve the problem. Interpret the solution and determine if the solution is reasonable. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 17 | Understand that polynomials are closed under the operations of addition, subtraction, and multiplication with integers; Add, subtract and multiply polynomials and divide polynomials by monomials. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 18 | Factor common terms from polynomials and factor polynomials completely. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 19 | Factor the difference of two squares, perfect square trinomials and other quadratic expression. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 20 | Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 21 | Graph and describe quadratic functions with and without technology. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 22 | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 23 | Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression. |
| Mathematics | Algebra 1 | Quadratics and Polynomials | 24 | Represent real-world problems using quadratic equations and solve such problems. Interpret the solution(s) and determine if the solution(s) is reasonable. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 1 | Functions | 25 | Rewrite square roots of non-perfect square integers and algebraic monomials |
| Mathematics | Algebra 1 | Functions | 26 | Use graphing technology to find approximate solutions of exponential and power functions. |
| Mathematics | Algebra 1 | Functions | 27 | Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, has a maximum or minimum value). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. Identify independent and dependent variables and make predictions about the relationship. |

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| Mathematics | Algebra 1 | Algebraic Rational Expressions | 28 | Rewrite algebraic rational expressions in equivalent forms (i.e. numerators and denominators are monomial expressions with integer exponents). | * limit |
| Mathematics | Algebra 1 | Algebraic Rational Expressions | 29 | Write and solve algebraic proportions that lead to a linear equation including real-world problems. | |
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| Mathematics | Algebra 1 | Data Analysis & Probability | 1 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | |
| Mathematics | Algebra 1 | Data Analysis & Probability | 2 | Use technology to write a linear function that represents data in a scatter plot representing a linear association. Interpret the slope and y-intercept in the context of the data. Compute (using technology) and interpret the correlation coefficient. | |
| Mathematics | Algebra 1 | Data Analysis & Probability | 3 | Distinguish between correlation and causation. Evaluate reports based on data by considering the source of the data, the design of the study, the way the data are analyzed and displayed and whether the report confuses correlation with causation. | |
| Mathematics | Algebra 1 | Data Analysis & Probability | 4 | Summarize categorical data for two categories, that has been collected or provided, in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations, trends in the data and answer questions about the data. | |
| Mathematics | Algebra 1 | Data Analysis & Probability | 5 | Organize, display and analyze univariate and bivariate data (e.g. using tables, line plots, histograms and box plots). Summarize the data using measures of center (e.g. mean, median) and spread (e.g, range, inter-quartile range, percentiles, variance). Understand the effects of outliers on the data. | |

# Geometry

| Content Area | Course | Strand | Number | Content Area Topic | |
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| Mathematics | Geometry | Proofs | 1 | Identify and give examples of undefined terms, axioms and postulates), and theorems, and inductive and deductive proofs. Describe the structure of and relationships within an axiomatic system (undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems). | * |
| Mathematics | Geometry | Proofs | 2 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | |
| Mathematics | Geometry | Proofs | 3 | State, use, and examine the validity of the converse, inverse, and contrapositive of “if – then” and “if and only if” statements. | |
| Mathematics | Geometry | Proofs | 4 | Understand the differences among supporting evidence, counterexamples and actual proofs. | C 1&3 |
| Mathematics | Geometry | Proofs | 5 | Develop geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. | |
| Mathematics | Geometry | Segments, Lines and Planes | 6 | Identify, justify and apply properties of planes. Describe the intersection of two or more geometric figures in the plane. | |
| Mathematics | Geometry | Construction Segments, Lines a | 7 | Define, identify, and construct with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) congruent segments and angles, angle bisectors, perpendicular bisectors, altitudes, medians, parallel and perpendicular lines, and congruent triangles. Explain and justify the process used. | |
| Mathematics | Geometry | Segments, Lines and Planes | 8 | Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints. | |
| Mathematics | Geometry | Segments, Lines and Planes | 9 | Develop the distance formula using the Pythagorem Theorem. Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems. Find measures of the sides of polygons in the coordinate plane; apply this technique to compute the perimeters and areas of polygons in real-world and mathematical problems. | |
| Mathematics | Geometry | Segments, Lines and Planes | 10 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. Prove and apply theorems involving segments divided proportionally. | |
| Mathematics | Geometry | Segments, Lines and Planes | 11 | Identify and apply properties of and theorems about parallel and perpendiculars, write equations of parallel and perpendicular lines, and develop geometric proofs involving the relationships between special pairs of angles formed by parallel lines and transversals and perpendicular lines. | |
| Mathematics | Algebra 2 | Segments, Lines and Planes | 12 | Prove the slope criteria for parallel and perpendicular lines and use them to solve real-world and mathematical problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | |

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| Mathematics | Geometry | Triangles | 13 | Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point, a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity, isosceles triangle theorem and its converse. | |
| Mathematics | Geometry | Triangles | 14 | Explain how the criteria for triangle congruence (ASA, SAS, ASA and SSS) follow from the definition of congruence in terms of rigid motions. | C 15&13? |
| Mathematics | Geometry | Triangles | 15 | Use properties of congruent and similar triangles to solve real-world and mathematical problems involving sides, perimeters, and areas of triangles. | |
| Mathematics | Geometry | Triangles | 16 | Given two figures, use the definition of similarity in terms of similarity transformations such as the AA criterion to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | |
| Mathematics | Geometry | Triangles | 17 | Construct the inscribed and circumscribed circles of a triangle (with or without technology), and prove properties of angles for a quadrilateral inscribed in a circle. | |
| Mathematics | Geometry | Triangles | 18 | Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem. | |
| Mathematics | Geometry | Triangles | 19 | State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. | |
| Mathematics | Geometry | Triangles | 20 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | |
| Mathematics | Geometry | Triangles | 21 | Use trigonometric ratios and the Pythagorean Theorem to solve real-world and mathematical problems in two- and three-dimensions. | |
| Mathematics | Geometry | Triangles | 22 | Use special right triangles (30° - 60° and 45° - 45°) to solve real-world and mathematical problems. | |
| Mathematics | Geometry | Quadrilaterals | 23 | Describe, classify, and understand relationships among the quadrilaterals convex, concave, and regular polygons, square, rectangle, rhombus, parallelogram, trapezoid, and kite. | |
| Mathematics | Geometry | Quadrilaterals | 24 | Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas. | |
| Mathematics | Geometry | Quadrilaterals | 25 | Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. | *27 |
| Mathematics | Geometry | Quadrilaterals | 26 | Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate). | *26 |
| Mathematics | Geometry | Quadrilaterals | 27 | Represent triangles and quadrilaterals in the coordinate plane and create proofs related to the figures (e.g. using knowledge of slopes, parallel and perpendicular lines, distance formula and the Pythagorean Theorem to classify the figures as isosceles, right, equilateral, square, rectangle, parallelogram, etc. ). | |
| Mathematics | Geometry | Transformations | 28 | Use geometric descriptions of rigid motions to transform figures and predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent. | |

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| Mathematics | Geometry | Transformations | 29 | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor. |
| Mathematics | Geometry | Polygons | 30 | Identify types of symmetry (i.e., line, point, rotational, self-congruencies) of polygons. |
| Mathematics | Geometry | Polygons | 31 | Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle. |
| Mathematics | Geometry | Construction Circles | 32 | Construct the circle that passes through three given points not on a line, construct tangents to circles, tangent line from a point outside a given circle to the circle. Justify the process used. |
| Mathematics | Geometry | Circles | 33 | Define , identify and use relationships among: radius, diameter, arc, measure of an arc, chord, secant, tangent, and congruent concentric circles. |
| Mathematics | Geometry | Circles | 34 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. |
| Mathematics | Geometry | Circles | 35 | Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. |
| Mathematics | Geometry | Polyhera | 36 | Describe relationships between the faces, edges, and vertices of polyhedra. Create a net for a given polyhedron. Describe the polyhedron that can be made from a given net (or pattern). |
| Mathematics | Geometry | Polyhera | 37 | Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres. Solve problems involving congruent and similar solids. |
| Mathematics | Geometry | Polyhera | 38 | Describe sets of points on spheres: chords, tangents, and great circles. |
| Mathematics | Geometry | Polyhera | 39 | Describe symmetries of geometric solids. |
| Mathematics | Geometry | Geometry | 40 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). |
| Mathematics | Geometry | Polyhedra | 41 | Solve real-world and mathematical problems involving volume and surface area of prisms, cylinders, cones, spheres, and pyramids including problems that involve algebraic expressions, e.g., determine the area of the base of a regular pyramid given the volume and the fact that the height is 5 more than 3 times the area of the base. |
| Mathematics | Geometry | Polyhedra | 42 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). |
| Mathematics | Geometry | Geometry | 43 | Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries. |

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| Mathematics | Geometry | Measurement | 1 | Find measures of interior and exterior angles of polygons, explain and justifying the method used. |
| Mathematics | Geometry | Measurement | 2 | Define, find, and use measures of circumference, arc length, and areas of circles and sectors, and arcs and related angles (central, inscribed, and intersections of secants and tangents). Use these measures to solve problems. |

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| Mathematics | Geometry | Measurement | 3 | Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas. Identify and know properties of congruent and similar solids. |

# Algebra 2

| Content Area | Course | Strand | Number | Content Area Topic |
|---|---|---|---|---|
| Mathematics | Algebra 2 | Functions | 1 | Determine whether a relation represented by a table, graph, verbal description, or equation is a function. Add, subtract, multiply, and divide pairs of functions. |
| Mathematics | Algebra 2 | Functions | 2 | Understand and interpret statements that use function notation in terms of a context. Relate the domain of a function to its graph and to the quantitative relationship it describes. |
| Mathematics | Algebra 2 | Absolute Value | 3 | Graph and solve absolute value linear equations and inequalities. |
| Mathematics | Algebra 2 | Systems | 4 | Solve systems of two or three linear equations in two or three variables algebraically. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 2 | Systems | 5 | Write a system of linear equations in three variables that represents a real-world problem and solve the problem. Interpret the solution and determine if the solution is reasonable. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 2 | Systems | 6 | Solve a system of equations consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x^2 + y^2 = 3. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 2 | Quadratics | 7 | Graph quadratic functions. Identify intercepts, zeros, domain and range, and lines of symmetry. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 2 | Quadratics | 8 | Derive the equation of a parabola given a focus and directrix. |
| Mathematics | Algebra 2 | Quadratics | 9 | Represent real-world problems using quadratic equations and solve such problems. Interpret the solution(s) and determine if the solution(s) is reasonable. Use graphing technology in situations that involve more complex numbers. |
| Mathematics | Algebra 2 | Quadratics | 10 | Know there is a complex number i such that i^2 = -1, and every complex number has the form a + bi with a and b real. Use the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |
| Mathematics | Algebra 2 | Quadratics | 11 | Recognize when the quadratic formula gives complex solutions. Solve quadratic equations with real coefficients that have complex solutions and write them as a ± bi for real numbers a and b. |
| Mathematics | Algebra 2 | Exponential and Linear | 12 | Distinguish between situations that can be modeled with linear functions and with exponential functions |
| Mathematics | Algebra 2 | Exponential and Linear | 13 | Compare properties of linear and exponential functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| Mathematics | Algebra 2 | Exponential | 14 | Graph exponential functions. Identify and describe features such as, intercepts, zeros, domain and range, and asymptotic and end behavior. Interpret the parameters in an exponential function in terms of a context. Use graphing technology in situations that involve more complex numbers. |

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| Mathematics | Algebra 2 | Exponential | 15 | Use the properties of exponents to transform and interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)12^t, y = (1.2)^t/10, and classify them as representing exponential growth or decay. |
| Mathematics | Algebra 2 | Exponential and Logarithmic | 16 | Compare exponential and logarithmic functions using graphing technology. |
| Mathematics | Algebra 2 | Exponential and Logarithmic | 17 | Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve mathmatical problems. |
| Mathematics | Algebra 2 | Exponential and Logarithmic | 18 | Solve real-world word problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. |
| Mathematics | Algebra 2 | Polynomials | 19 | Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression. |
| Mathematics | Algebra 2 | Polynomials | 20 | Solve real-world word problems that can be represented using polynomial equations. Interpret the solutions and determine whether the solutions are reasonable. |
| Mathematics | Algebra 2 | Polynomials | 21 | Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle |
| Mathematics | Algebra 2 | Rational and Radical | 22 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| Mathematics | Algebra 2 | Rational and Radical | 23 | Rewrite rational expressions in equivalent forms, e.g., using properties of exponents and factoring techniques. |
| Mathematics | Algebra 2 | Rational and Radical | 24 | Rewrite rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using long division. |
| Mathematics | Algebra 2 | Rational and Radical | 25 | Relate expressions containing rational exponents to the corresponding radical expressions |
| Mathematics | Algebra 2 | Functions | 26 | Graph rational functions using technology. Identify and describe features such as, intercepts, domain and range, and asymptotic and end behavior. |
| Mathematics | Algebra 2 | Rational and Radical | 27 | Solve real-world and mathematical problems involving rational and radical equations, inculding direct, inverse, and joint variation. Give examples showing how extraneous solutions may arise. |
| Mathematics | Algebra 2 | Functions | 28 | Graph relations and functions including polynomial, square root, cube root, and piecewise-defined functions (including step functions and absolute value functions). Identify and describe features such as, intercepts, zeros, domain and range, end behavior, and lines of symmetry. Use graphing technology in situations that involve more complex numbers and to approximate solutions of the equations. |

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| Mathematics | Algebra 2 | Data Analysis & Probability | 1 | Analyze data to determine if it suggests a linear, quadratic, or exponential relationship. Use technology to write a function to represent the data and solve problems using the function. |
| Mathematics | Algebra 2 | Data Analysis & Probability | 2 | Use simulations to construct empirical probability distributions. |
| Mathematics | Algebra 2 | Data Analysis & Probability | 3 | Understand the concepts of conditional probability and independent events. |
| Mathematics | Algebra 2 | Data Analysis & Probability | 4 | Understand the counting principle, permutations, and combinations and use them to solve contextual word problems |
| Mathematics | Algebra 2 | Data Analysis & Probability | 5 | Define arithmetic and geometric sequences and series, find specified terms of arithmetic and geometric sequences |
| Mathematics | Algebra 2 | Data Analysis & Probability | 6 | Find partial sums of arithmetic and geometric series |
| Mathematics | Algebra 2 | Data Analysis & Probability | 7 | Solve contextual word problems involving applications of sequences and series, write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions |

# Pre-Calculus

| Content Area | Course | Strand | Number | Content Area Topic |
|---|---|---|---|---|
| Mathematics | Pre-Calculus | Polynomials | 1 | Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). |
| Mathematics | Pre-Calculus | Quadratic | 2 | Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. |
| Mathematics | Pre-Calculus | Functions | 3 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity |
| Mathematics | Pre-Calculus | Functions | 4 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers |
| Mathematics | Pre-Calculus | Functions | 5 | Understand composition of functions and combine functions by composition. |
| Mathematics | Pre-Calculus | Functions | 6 | Find and verify by composition that one function is the inverse of another. |
| Mathematics | Pre-Calculus | Functions | 7 | Produce an invertible function from a non-invertible function by restricting the domain |
| Mathematics | Pre-Calculus | Functions | 8 | Find a quadratic, exponential, logarithmic, power, or sinusoidal function to model a data set and explain the parameters of the model. |
| Mathematics | Pre-Calculus | Functions | 9 | Determine if a graph or table has and inverse and if it has an inverse is the inverse a function or relation. Identify the values of an inverse function/relation from a graph or a table, given that the function has an inverse. Justify the values are on the inverse function/relation. Derive the inverse equation from the values of the inverse. |
| Mathematics | Pre-Calculus | Exponential | 10 | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another |
| Mathematics | Pre-Calculus | Logarithms | 11 | Use the definition of logarithms to convert logarithms from one base to another, prove simple laws of logarithms. |
| Mathematics | Pre-Calculus | Logarithms | 12 | Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values. |
| Mathematics | Pre-Calculus | Exponential and Logarithmic | 13 | For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. |
| Mathematics | Pre-Calculus | Exponential and Logarithmic | 14 | Solve logarithmic and exponential equations and inequalities. |
| Mathematics | Pre-Calculus | Functions | 15 | Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x),f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them |
| Mathematics | Pre-Calculus | Conics | 16 | Determine how the graph of a parabola changes if a, b and c changes in the equation y = a(x – b)2 + c. Find an equation for a parabola when given sufficient information |

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| Mathematics | Pre-Calculus | Conics | 17 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation |
| Mathematics | Pre-Calculus | Conics | 18 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant |
| Mathematics | Pre-Calculus | Conics | 19 | Graph conic sections. Identify and describe features like center, vertex or vertices, focus, directrix, axis of symmetry, major axis, minor axis, and eccentricity. |
| Mathematics | Pre-Calculus | Complex Numbers | 20 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. |
| Mathematics | Pre-Calculus | Complex Numbers | 21 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. |
| Mathematics | Pre-Calculus | Complex Numbers | 22 | Represent addition, subtraction, multiplication, and conjugation of complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. |
| Mathematics | Pre-Calculus | Algebra & Functions | 23 | State, prove, and use De Moivre's Theorem |
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| Mathematics | Pre-Calculus | Geometry | 1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments |
| Mathematics | Pre-Calculus | Geometry | 2 | Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane |
| Mathematics | Pre-Calculus | Geometry | 3 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects |
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| Mathematics | Pre-Calculus | Data Analysis & Probability | 1 | Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge. Recognize an infinite series as the limit of a sequence of partial sums |
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| Mathematics | Pre-Calculus | Calculus | 1 | Decide if a function is continuous at a point, understand continuity in terms of limits |
| Mathematics | Pre-Calculus | Calculus | 2 | Find the types of discontinuities of a function and relate them to fininding limits of a function. |
| Mathematics | Pre-Calculus | Calculus | 3 | Understand the concept of limit and estimate limits from graphs and tables of values |
| Mathematics | Pre-Calculus | Calculus | 4 | Find limits at infinity |

# Trigonometry

| Content Area | Course | Strand | Number | Content Area Topic |
|---|---|---|---|---|
| Mathematics | Trigonometry | Unit Circle | 1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle |
| Mathematics | Trigonometry | Unit Circle | 2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle |
| Mathematics | Trigonometry | Unit Circle | 3 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions |
| Mathematics | Trigonometry | Functions | 4 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline |
| Mathematics | Trigonometry | Functions | 5 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed |
| Mathematics | Trigonometry | Functions | 6 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context |
| Mathematics | Trigonometry | Identities | 7 | Prove the Pythagorean identity sin2(?) + cos2(?) = 1 and use it to find sin(?), cos(?), or tan(?) given sin(?), cos(?), or tan(?) and the quadrant of the angle |
| Mathematics | Trigonometry | Algebra & Functions | 8 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems |
| Mathematics | Trigonometry | Algebra & Functions | 9 | Prove the double- and half-angle formulas for sine, cosine, and tangent and use them to solve problems |
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| Mathematics | Trigonometry | Geometry | 1 | Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles |
| Mathematics | Trigonometry | Geometry | 2 | Solve contextual problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate |
| Mathematics | Trigonometry | Geometry | 3 | Explain and use the relationship between the sine and cosine of complementary angles |
| Mathematics | Trigonometry | Geometry | 4 | Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4 and p/6, and use the unit circle to express the values of sine, cosine, and tangent for x, p + x, and 2p – x in terms of their values for x, where x is any real number |
| Mathematics | Trigonometry | Geometry | 5 | Prove the Laws of Sines and Cosines and use them to solve problem |
| Mathematics | Trigonometry | Geometry | 6 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces) |
| Mathematics | Trigonometry | Geometry | 7 | Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side |

# Finite Math

| Content Area | Course | Number | Content Area Topic |
|---|---|---|---|
| Mathematics | Finite Math | 1 | Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexamples in mathematical reasoning. Construct logical arguments, assess the validity of logical arguments and give counterexamples to disprove statements |
| Mathematics | Finite Math | 2 | Use mathematical induction to prove simple propositions |
| Mathematics | Finite Math | 3 | Recognize syllogisms, tautologies, flawed reasoning and circular reasoning |
| Mathematics | Finite Math | 4 | Know and use the concepts of sets, elements and subsets |
| Mathematics | Finite Math | 5 | Perform operations on sets (union, intersection, complement, cross product) |
| Mathematics | Finite Math | 6 | Explore function iteration and, in the process, informally introduce function composition |
| Mathematics | Finite Math | 7 | Use networks, traceable paths, tree diagrams, Venn diagrams, and other pictorial representations to find the number of outcomes in a problem situation |
| Mathematics | Finite Math | 8 | Optimize networks in different ways and in different contexts by finding minimal spanning trees, shortest paths, and Hamiltonian paths |
| Mathematics | Finite Math | 9 | Use critical-path analysis to solve scheduling problems |
| Mathematics | Finite Math | 10 | Construct and interpret directed and undirected graphs, decision trees, networks and flow charts |
| Mathematics | Finite Math | 11 | Understand matrices as systems that have some of the properties of the real-number system. |
| Mathematics | Finite Math | 12 | Multiply matrices by scalars to produce new matrices. |
| Mathematics | Finite Math | 13 | Add, subtract, and multiply matrices of appropriate dimensions (i.e. up to 3x3 matrices). |
| Mathematics | Finite Math | 14 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. |
| Mathematics | Finite Math | 15 | Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems. |
| Mathematics | Finite Math | 16 | Use matrices to solve real-world problems that can be modeled by a system of equations (i.e. up to 3 linear equations) in two or three variables using technology. |
| Mathematics | Finite Math | 17 | Use an adjacency matrix to describe a vertex-edge graph |
| Mathematics | Finite Math | 18 | Perform row and column sums for matrix equations |
| Mathematics | Finite Math | 19 | Build and use matrix representations to model polygons, transformations, and computer animations |
| Mathematics | Finite Math | 20 | Understand vectors as systems that have some of the properties of the real-number system |
| Mathematics | Finite Math | 21 | Solve problems involving velocity and other quantities that can be represented by vectors |
| Mathematics | Finite Math | 22 | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy) |
| Mathematics | Finite Math | 23 | Compute the magnitude of a scalar multiple cv using \|\|cv\|\| = \|c\|v. Compute the direction of cv knowing that when \|c\|v ? 0, the direction of cv is either along v (for c > 0) or against v (for c < 0) |
| Mathematics | Finite Math | 24 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors |
| Mathematics | Finite Math | 25 | Use graph-coloring techniques to solve problems |
| Mathematics | Finite Math | 26 | Use bin-packing techniques to solve problems of optimizing resource usage |
| Mathematics | Finite Math | 27 | Convert between a pair of parametric equations and an equation in x and y |
| Mathematics | Finite Math | 28 | Analyze planar curves, including those given in parametric form |
| Mathematics | Finite Math | 29 | Model and solve problems using parametric equations |

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| Mathematics | Finite Math | 30 | Use row-reduction techniques to solve problems |
| Mathematics | Finite Math | 31 | Use Markov chains to solve problems |
| Mathematics | Finite Math | 32 | Use finite differences to solve problems |
| Mathematics | Finite Math | 33 | Use graphs consisting of vertices and edges to model a problem situation |
| Mathematics | Finite Math | 34 | Use minimal spanning trees to solve problems |
| Mathematics | Finite Math | 35 | Use geometric techniques to solve optimization problems |
| Mathematics | Finite Math | 36 | Use the Simplex method to solve optimization problems with and without technology |

# Probability & Statistics

| Content Area | Course | Number | Content Area Topic |
|---|---|---|---|
| Mathematics | Probability & Statistics | 1 | Create, compare, and evaluate different graphic displays of the same data, using histograms, frequency polygons, cumulative frequency distribution functions, pie charts, scatterplots, stem-and-leaf plots, and box-and-whisker plots. Draw these by hand or use a computer spreadsheet program |
| Mathematics | Probability & Statistics | 2 | Compute and use mean, median, mode, weighted mean, geometric mean, harmonic mean, range, quartiles, variance, and standard deviation |
| Mathematics | Probability & Statistics | 3 | Define and use the mathematical induction method of proof |
| Mathematics | Probability & Statistics | 4 | Understand the central limit theorem and use it to solve problems |
| Mathematics | Probability & Statistics | 5 | Compute and use confidence intervals to make estimates |
| Mathematics | Probability & Statistics | 6 | Construct and interpret margin of error and confidence intervals for population proportions |
| Mathematics | Probability & Statistics | 7 | Compute and interpret the expected value of random variables in simple cases |
| Mathematics | Probability & Statistics | 8 | Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events |
| Mathematics | Probability & Statistics | 9 | Understand and use the multiplication rule to calculate probabilities for independent and dependent events |
| Mathematics | Probability & Statistics | 10 | Use counting techniques to solve probability problems |
| Mathematics | Probability & Statistics | 11 | Calculate the probabilities of complementary events |
| Mathematics | Probability & Statistics | 12 | Understand the conditional probability of A given B as P(A andB)/P(B), and interpret independence of A and B as saying that the conditional probability of A givenB is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B and use Bayes' Theorem and use them to solve problems |
| Mathematics | Probability & Statistics | 13 | Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events |
| Mathematics | Probability & Statistics | 14 | Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent |
| Mathematics | Probability & Statistics | 15 | Calculate the expected value of a random variable; interpret it as the mean of the probability distribution |
| Mathematics | Probability & Statistics | 16 | Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) |
| Mathematics | Probability & Statistics | 17 | Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions |
| Mathematics | Probability & Statistics | 18 | Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values |
| Mathematics | Probability & Statistics | 19 | Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value |
| Mathematics | Probability & Statistics | 20 | Use simulations to solve counting and probability problems |
| Mathematics | Probability & Statistics | 21 | Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples |
| Mathematics | Probability & Statistics | 22 | Recognize how linear transformations of univariate data affect shape, center, and spread |
| Mathematics | Probability & Statistics | 23 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities |
| Mathematics | Probability & Statistics | 24 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation |
| Mathematics | Probability & Statistics | 25 | Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable |
| Mathematics | Probability & Statistics | 26 | Identify, display, and discuss trends in bivariate data and find functions that model the data or transform the data so that they can be modeled |
| Mathematics | Probability & Statistics | 27 | Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results |
| Mathematics | Probability & Statistics | 28 | Evaluate information based on data by considering the source of the data, the design of the study, the way the data are analyzed and displayed, and whether the report confuses correlation with causation |
| Mathematics | Probability & Statistics | 29 | Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them |
| Mathematics | Probability & Statistics | 30 | Select and use appropriate statistical methods to analyze data |
| Mathematics | Probability & Statistics | 31 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling |
| Mathematics | Probability & Statistics | 32 | Use election theory techniques to analyze election data |
| Mathematics | Probability & Statistics | 33 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve |
| Mathematics | Probability & Statistics | 34 | Analyze and apply algorithms for searching (sequential, binary), for sorting (bubble sort, quick sort, bin sort) and for solving optimization problems |

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| Mathematics | Probability & Statistics | 35 | Analyze and interpret relationships defined iteratively and recursively. Use recursive thinking to solve problems |
| Mathematics | Probability & Statistics | 36 | Define arithmetic and geometric sequences recursively. Use a variety of recursion equations to describe a function. |
| Mathematics | Probability & Statistics | 37 | Construct simulated sampling distributions of sample proportions and use sampling distributions to identify which proportions are likely to be found in a sample of a given size |
| Mathematics | Probability & Statistics | 38 | Construct vertex-edge graph models involving relationships among a finite number of elements |
| Mathematics | Probability & Statistics | 39 | Derive the binomial theorem by combinatorics |
| Mathematics | Probability & Statistics | 40 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”) |
| Mathematics | Probability & Statistics | 41 | Determine the number of ways events can occur using permutations, combinations and the Fundamental Counting Principle |
| Mathematics | Probability & Statistics | 42 | Determine whether two propositions are logically equivalent |
| Mathematics | Probability & Statistics | 43 | Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving |
| Mathematics | Probability & Statistics | 44 | Distinguish between inductive and deductive reasoning. Identify inductive reasoning as central to the scientific method and deductive reasoning as characteristic of mathematics |
| Mathematics | Probability & Statistics | 45 | Experience in mathematical modeling by building and using vertex-edge graph models to solve problems in a variety of real-world settings |
| Mathematics | Probability & Statistics | 46 | Explore the geometric, or waiting-time, distribution |
| Mathematics | Probability & Statistics | 47 | Find linear models by using median fit and least squares regression methods. Decide which among several linear models gives a better fit. Interpret the slope in terms of the original context |
| Mathematics | Probability & Statistics | 48 | Informally assess the fit of a function by plotting and analyzing residuals |
| Mathematics | Probability & Statistics | 49 | Make predictions from the least squares regression line or its equation |
| Mathematics | Probability & Statistics | 50 | Model and solve problems involving patterns using recursion and iteration, growth and decay, and compound interest |
| Mathematics | Probability & Statistics | 51 | Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable |
| Mathematics | Probability & Statistics | 52 | Understand and apply basic ideas related to the design and interpretation of surveys, such as background information, random sampling, and bias |
| Mathematics | Probability & Statistics | 53 | Understand and apply recursion equations, particularly combined recursion equations of the form An = rAn-1 + b. |
| Mathematics | Probability & Statistics | 54 | Understand how basic statistical techniques are used to monitor process characteristics in the workplace |
| Mathematics | Probability & Statistics | 55 | Understand how sample statistics reflect the values of the population parameters and use sampling distributions as the basis for informal inference |
| Mathematics | Probability & Statistics | 56 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population |
| Mathematics | Probability & Statistics | 57 | Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each |
| Mathematics | Probability & Statistics | 58 | Use a recursion function to describe a fractal |
| Mathematics | Probability & Statistics | 59 | Use and interpret relational conjunctions (and, or, not), terms of causation (if… then) and equivalence (if and only if). Distinguish between the common uses of such terms in everyday language and their use in mathematics |
| Mathematics | Probability & Statistics | 60 | Use combinatorial reasoning to solve problems |
| Mathematics | Probability & Statistics | 61 | Use fair division techniques to solve apportionment problems and to divide continuous objects |
| Mathematics | Probability & Statistics | 62 | Use game theory to solve strictly and non strictly determined games |
| Mathematics | Probability & Statistics | 63 | Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change |
| Mathematics | Probability & Statistics | 64 | Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions |
| Mathematics | Probability & Statistics | 65 | Use truth tables to determine the truth values of propositional statements |
| Mathematics | Probability & Statistics | 66 | Use weighted voting techniques to decide voting power within a group |

# Calculus

| Content Area | Course | Number | Content Area Topic |
|---|---|---|---|
| Mathematics | Calculus | 1 | Find the types of discontinuities of a function |
| Mathematics | Calculus | 2 | Decide if a function is continuous at a point |
| Mathematics | Calculus | 3 | Understand the concept of limit and estimate limits from graphs and tables of values |
| Mathematics | Calculus | 4 | Understand continuity in terms of limits |
| Mathematics | Calculus | 5 | Find limits of rational functions that are undefined at a point |
| Mathematics | Calculus | 6 | Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior |
| Mathematics | Calculus | 7 | Find limits at infinity |
| Mathematics | Calculus | 8 | Find limits by substitution |
| Mathematics | Calculus | 9 | Find limits of sums, differences, products, and quotients |
| Mathematics | Calculus | 10 | Find one-sided limits |
| Mathematics | Calculus | 11 | Understand and use the Intermediate Value Theorem on a function over a closed interval |
| Mathematics | Calculus | 12 | Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval |
| Mathematics | Calculus | 13 | Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change |
| Mathematics | Calculus | 14 | Find local and absolute maximum and minimum points |
| Mathematics | Calculus | 15 | Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f". Understand points of inflection as places where concavity changes |
| Mathematics | Calculus | 16 | Find a tangent line to a curve at a point and a local linear approximation |
| Mathematics | Calculus | 17 | Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents |
| Mathematics | Calculus | 18 | State, understand, and apply the definition of derivative |
| Mathematics | Calculus | 19 | Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions |
| Mathematics | Calculus | 20 | Find the derivatives of sums, products, and quotients |
| Mathematics | Calculus | 21 | Find the derivatives of composite functions, using the chain rule |
| Mathematics | Calculus | 22 | Find the derivatives of implicitly-defined functions |
| Mathematics | Calculus | 23 | Find derivatives as inverse functions |
| Mathematics | Calculus | 24 | Find second derivatives and derivatives of higher order |
| Mathematics | Calculus | 25 | Find derivatives using logarithmic differentiation |
| Mathematics | Calculus | 26 | Understand and use the relationship between differentiability and continuity |
| Mathematics | Calculus | 27 | Understand and apply the Mean Value Theorem |
| Mathematics | Calculus | 28 | Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f' |
| Mathematics | Calculus | 29 | Analyze curves, including the notions of monotonicity and concavity |
| Mathematics | Calculus | 30 | Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of f, f', and f" |
| Mathematics | Calculus | 31 | Use implicit differentiation to find the derivative of an inverse function |
| Mathematics | Calculus | 32 | Solve optimization problems |

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| Mathematics | Calculus | 33 | Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration |
| Mathematics | Calculus | 34 | Find the velocity and acceleration of a particle moving in a straight line |
| Mathematics | Calculus | 35 | Model rates of change, including related rates problems |
| Mathematics | Calculus | 36 | Use rectangle approximations to find approximate values of integrals. |
| Mathematics | Calculus | 37 | Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points |
| Mathematics | Calculus | 38 | Interpret a definite integral as a limit of Riemann Sums. |
| Mathematics | Calculus | 39 | Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $f'(x)dx = f(b) - f(a)$. |
| Mathematics | Calculus | 40 | Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined |
| Mathematics | Calculus | 41 | Understand and use these properties of definite integrals |
| Mathematics | Calculus | 42 | Understand and use integration by substitution (or change of variable) to find values of integrals |
| Mathematics | Calculus | 43 | Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values |
| Mathematics | Calculus | 44 | Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line |
| Mathematics | Calculus | 45 | Solve separable differential equations and use them in modeling |
| Mathematics | Calculus | 46 | Solve differential equations of the form $y' = ky$ as applied to growth and decay problems. |
| Mathematics | Calculus | 47 | Use definite integrals to find the area between a curve and the x-axis, or between two curves |
| Mathematics | Calculus | 48 | Use definite integrals to find the average value of a function over a closed interval |
| Mathematics | Calculus | 49 | Use definite integrals to find the volume of a solid with known cross-sectional area |
| Mathematics | Calculus | 50 | Apply integration to model and solve problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral |